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CALIBRATING THE CREDITMETRICS™ CORRELATION CONCEPT – EMPIRICAL EVIDENCE FROM GERMANY

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1. Introduction

Among the major challenges of credit risk measurement is the issue of modelling the joint default behaviour in a portfolio of fixed-income securities, e. g. corporate bonds or loans. Ignoring the impact of up- or downgrades in either the rating agencies' external or banks' own internal rating systems on the securities' market values and focussing instead on a hold-to-maturity point of view, a proper credit risk measurement must, in principle, quantify the probabilities of joint default events across all obligors for the relevant risk horizon. Rooted in accounting practice, the time horizon in credit risk models is usually one year.

Whereas a probability of default (PD) is comparatively easy to estimate for a single obligor firm, it is almost impossible directly to estimate probabilities for the joint default events in a loan portfolio comprising several hundreds or thousands of companies.[1] This problem arises because of the sheer number of probabilities: For a portfolio with n obligors, the bank's internal or other external rating systems should provide n PDs, giving the probability for each firm that this particular firm will default on one of its obligations during the year to come, regardless of what happens to the other $n - 1$ firms. However, in order to obtain a picture of the entire portfolio loss distribution, one would have to estimate the 2^n probabilities of all possible joint default events. In order to provide a workable approximation of the complete loss distribution, several models of credit portfolio risk have been developed in the banking industry.

The origins of quantitative credit risk modelling at the portfolio level can be traced back to the year 1997, when J.P. MORGAN (in conjunction with several co-sponsors) launched CreditMetrics™ and CREDIT SUISSE FIRST BOSTON released its CreditRisk$^+$.[2] The widely used CreditMetrics™ software implementation CreditManager™ is now marketed by RiskMetricsGroup™, a J.P. MORGAN spin-off. Together with KMV's

Portfolio Manager™ and MCKINSEY's Credit-PortfolioView™, these two portfolio models still form the cornerstone of current industry practice[3] and their underlying concepts are at the center of ongoing theoretical debate. CreditMetrics™ and the KMV model[4] are both asset value approaches, sometimes also referred to as "structural" – in contrast to "reduced-form" – models.[5] Their common notion that default is triggered if the market value of the firm's total assets falls below some critical threshold level, derives from the seminal work of MERTON (1974). Not only the two widely-used vendor models, but also many of today's *internal* credit portfolio models are based on this approach and, since most of the financial institutions are heavily involved in giving loans to non-publicly traded firms, they all need to find a workable solution to the problem addressed in this paper.

This paper deals with the problem of empirically calibrating the CreditMetrics™ correlation concept for a portfolio of loans to non-publicly-traded German firms.[6] Using the CreditMetrics™ framework to determine the probabilities of joint default events requires, for each obligor, an estimation of the portion of asset return volatility that is firm-specific (idiosyncratic). From a practitioner perspective, estimating these weights is a very important, probably the *most sensitive* calibration problem in credit portfolio modelling. Setting the percentage portion of idiosyncratic risk has a *tremendous* impact on the resulting loss distribution, especially on its lower tail. A comparatively small variation in these weights can lead to a huge increase in the Credit Value at Risk for a "typical" loan portfolio with the Credit Value at Risk usually defined as some quantile of the distribution (e. g. 99,95%) less its expected loss. Because of an almost total lack of academic literature, this calibration problem seems to be the one that is least understood by CreditMetrics™ users and it involves the hardest "guess" for them. Although the problem lies at heart of all asset correlation concepts in credit portfolio modelling, there is, as far as we know, just one empirical study on this subject which was carried out for internal purposes by the RiskMetricsGroup™ themselves.[7]

The article is organized as follows. In section 2, we introduce a restricted version of the CreditMetrics™ framework similar to that used by GORDY (2000), p. 124 f. We then present a simplified version of the index model that accounts for the reduced data requirements necessary for medium-sized enterprises[8] and we briefly review the RiskMetricsGroup™ approach to the estimation of a non-listed company's systematic risk. In section 3, we provide a detailed motivation for our own empirical study that uses 55 weekly stock returns for 250 randomly selected German stocks and we introduce our two-step regression methodology. In section 4, we investigate, whether or not the inclusion of our two alternative proxy variables for company size improves the predictive power of the CreditMetrics™ index model with respect to pairwise asset correlations. Section 5 summarizes our findings and considers issues for future research.

2. The CreditMetrics™ Methodology Revisited

2.1 Deriving the Probability Distribution of Portfolio Loss from Asset Correlations

Consider a bank's loan portfolio with n different corporate obligors. Each obligor firm i is characterized by its (strictly positive) probability of default PD_i, which can be regarded as inferred from the bank's rating systems.[9] We further assume that each company's asset return $\tilde{r}_i$ obeys a standard normal distribution. The asset return, which is in fact a latent variable, can be regarded as describing the annual percentage change in the market value of the firm's total assets. It is a one-period measure of the overall corporate business performance. The standard normal distribution is characterized by its probability density function φ or its cumulative probability density function Φ.

Company i defaults, if and only if its realized asset return r_i in the ensuing year falls below the critical level r_i', the so-called default threshold:

$$PD_i = P\{\tilde{r}_i < r_i'\} = \int_{-\infty}^{r_i'} \varphi(r_i)\ dr_i = \Phi(r_i')$$
$$\Leftrightarrow r_i' = \Phi^{-1}(PD_i). \qquad (1)$$

Obviously, the "cut-off" return r_i' is a function of the company's PD, which may, in turn, be derived from the firm's rating class. Note that there is one important difference between this and the usual framework of option pricing theory[10]: Here, the firms' PDs are given *exogeniously*, for example by the bank's rating system, so that there is no need to estimate the volatilities of the n asset returns. Furthermore, the assumption of all asset returns being normally distributed with mean zero and standard deviation one is far from being as severe as it may seem at first glance. In fact, a change of these parameters would only result in a set of default thresholds differing from that given by equation (1). In this sense, the distributional assumption can be made without loss of generality. For ease of exposition, we now turn to the case of a simple two-obligor-portfolio. The random asset returns $\tilde{r}_i$ and $\tilde{r}_j$ are assumed to be drawn from a *bivariate* standard normal distribution with joint density function φ_2 and known correlation coefficient ρ_{ij}[11]:

$$\varphi_2(r_i; r_j; \rho_{ij}) = \frac{1}{2\pi\sqrt{1-\rho_{ij}^2}} \exp\left(-\frac{1}{2(1-\rho_{ij}^2)}(r_i^2 - 2\rho_{ij} r_i r_j + r_j^2)\right). \qquad (2)$$

Thus, the shape of the joint asset return density function of two firms is described completely by the single parameter ρ_{ij}:

$$\rho_{ij} = \frac{Cov(\tilde{r}_i; \tilde{r}_j)}{\sqrt{Var(\tilde{r}_i)} \cdot \sqrt{Var(\tilde{r}_j)}}. \qquad (3)$$

The coefficient ρ_{ij} represents the asset return correlation between the companies i and j and is referred to in short as *asset correlation*. It is a measure of the co-movement of their asset returns which in turn reflect their business success. Starting with MARKOWITZ' (1952) theory of portfolio selection, the notion of correlation itself has, for decades, not been questioned as to its suitability for applications in financial risk management. Although correlation only reflects the linear dependence between two random variables and there is a growing body of literature dealing with the shortcomings of this measure[12], it is still the predominant paradigm in most practical issues of credit portfolio risk management.

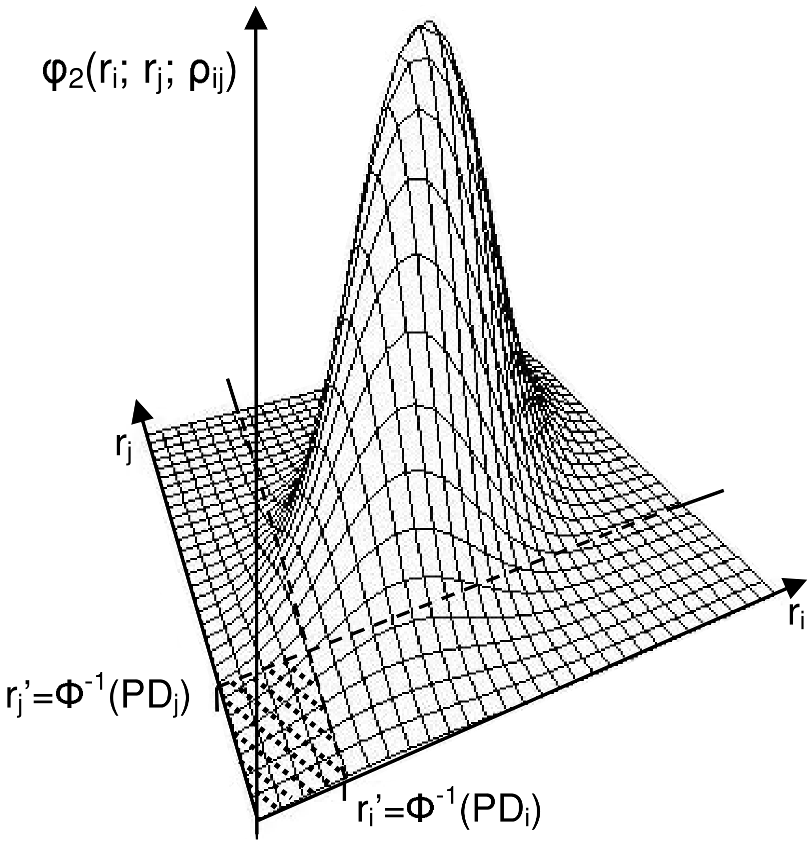
The probability that both obligors i and j will default *jointly* is denoted by PD_{ij}. This probability is calculated using the default thresholds r_i' and r_j' which result from PD_i and PD_j via Eq. (1) together with the bivariate asset return density function (2)[13]:

$$PD_{ij} = P\{\tilde{r}_i < r_i' \wedge \tilde{r}_j < r_j'\}$$
$$= \int_{-\infty}^{\Phi^{-1}(PD_i)} \int_{-\infty}^{\Phi^{-1}(PD_j)} \varphi_2(r_i; r_j; \rho_{ij})\, dr_i dr_j \qquad (4)$$
$$= \Phi_2\left(\Phi^{-1}(PD_i); \Phi^{-1}(PD_j); \rho_{ij}\right).$$

The bivariate standard normal ("Gaussian") distribution function Φ_2 can be interpretated as the CreditMetrics™ *copula* function.[13] A graphic illustration of the relationship between the joint density function of asset returns, the default thresholds and the joint probability of default is given in Figure 1, assuming an asset correlation of $\rho_{ij} = 0{,}6$.

The area that contains those combinations of r_i and r_j that lead to a joint default of both obligors lies in the lower left corner of the base and is hatched. The corresponding probability PD_{ij} is the volume of the solid area between the hatched part of the $(r_i;\ r_j)$-plane and the bivariate density function (2). With the help of Figure 1, it is easy to illustrate the fact that the probability of a joint

Figure 1: Relationship Between the Bivariate Standard Normal Density Function of Asset Returns, the Default Thresholds and the Joint Probability of Default for ρ_{ij} = 0,6.



default PD_{ij} is a strictly monotonously increasing function of the asset correlation and also rises strictly with an increasing PD. Having clarified the relationship between asset correlations and joint PDs in the CreditMetrics™ framework, we now turn to the calculation of the *portfolio loss distribution*. Let us assume that the bank has an exposure of E_i dollars to obligor i and of E_j dollars to obligor j. In default, the recovery rates are given as a percentage of the respective exposure and denoted by RR_i and RR_j. We deliberately exclude the stochastic recovery rate available in CreditMetrics™, as this would complicate our analysis unnecessarily. Hence, the probability distribution of portfolio loss comprises only the following four states: Either neither of the two obligors defaults (state 1), both default (state 2) or just one defaults (state 3 for obligor i and state 4 for j). The calculation of the probabilities of these four states p(s) and the corresponding portfolio losses L(s) is now straightforward, because the probability of a joint default $p(2) = PD_{ij}$ is given by Eq. (4) and we also know from Eq. (1) that $p(3) = PD_j - PD_{ij}$ and $p(4) = PD_i - PD_{ij}$. Hence, the probability distribution of the potential losses in the loan portfolio is characterized completely by the individual PDs of the obligors together with their asset correlation. This concept, which can be generalized from our simple two-obligor-illustration to the case of an n obligor loan portfolio[15], forms the core of the CreditMetrics™ asset correlation approach to the modelling of joint default events.

2.2 Estimating Asset Correlations by means of an Index Model

Determining the loss distribution for a portfolio of n obligors as described above, requires empirical estimates of the $n \cdot (n - 1) / 2$ pairwise asset correlations for the ensuing year. In order to reduce data requirements and simplify the parameter estimation, the CreditMetrics™ methodology deduces estimates of the obligors' individual asset correlations from stock indices by means of a factor model.[16] In the general CreditMetrics™ approach, both country and industry weights are assigned to each obligor according to its participation.[17] For our purposes, we make the following two simplifying assumptions. Firstly, we ignore potential calibration problems arising from *cross-country diversification*. Since our focus is on a portfolio of German non-listed corporate obligors, all country weights can simply be set to 100% for Germany, so that all other countries are ignored. Hence, the degree of concentration in such a purely national loan portfolio is driven mainly by the companies' industry composition. Secondly, we do not explicitly consider those calibration problems that concern *conglomerates*. The internal databases of banks often offer just one industry affiliation per customer, but for different industry classification systems, such as the WZ93 code of the Statistisches Bundesamt or according to MOODY'S industry group code. A percentage allocation of one customer to more than one industry, which is available in CreditManager™ and offered by REUTERS or BLOOMBERG for many publicly-traded companies, is therefore not feasible for most medium-sized corporate obligors. Consequently, all that can be achieved is to map each firm i = 1..n to its affiliated industry k(i) = 1..m (m << n). With these simplifycations, the CreditMetrics™ index model can be expressed as:

$$\tilde{r}_i = \sqrt{w_i} \cdot \tilde{f}_{k(i)} + \sqrt{1 - w_i} \cdot \tilde{\varepsilon}_i, \ \forall \ i = 1..n, \qquad (5)$$

with

$$w_i \in [0;1], \quad \forall\, i = 1..n,$$

$$\tilde{r}_i, \tilde{f}_{k(i)}, \tilde{\varepsilon}_i \sim N(0;1), \quad \forall\, i = 1..n,$$

$$Cov(\tilde{\varepsilon}_i; \tilde{\varepsilon}_j) = 0, \quad \forall\, i = 1..n,\ j = 1..n, i \neq j,$$

$$Cov(\tilde{\varepsilon}_i; \tilde{f}_{k(j)}) = 0, \quad \forall\, i = 1..n,\ j = 1..n.$$

$\tilde{f}_{k(i)}$ denotes the return of the industry index k to which company i is classified. A list of the indices which have been actually used for Germany in the first version of CreditManager™ can be found in Appendix I of the Technical Document.[18] The company-specific noise term $\tilde{\varepsilon}_i$ represents the *idiosyncratic* (also referred to as firm-specific or unsystematic) movements in asset returns. Each obligor's noise term is assumed to be uncorrelated with the noise terms of all other firms and furthermore as uncorrelated with the movements that affect the industry as a whole and which are therefore fully reflected in the respective index return. We use the term idiosyncratic risk in the sense that it describes that component of the total variation in the asset return of a company which cannot be explained by its industry affiliation. Correspondingly, we refer to the industry influence embodied in the movements of the respective index as *systematic*, because these movements can be regarded as induced by changes in latent variables which affect many firms (at least more than one firm) in that particular industry.[19]

w_i and $1 - w_i$ represent the weights that must be assigned to the industry or alternatively to the firm-specific influence on asset returns. The greater w_i, the closer the firm tracks its industry performance and the less it moves independently of its industry associates. The weights are scaled in such a way that all random variables can be modelled as standard normally distributed.[20] As a result, it is possible to divide the total risk in fluctuating asset returns in two different components which do not intersect:

$$\underbrace{\underbrace{\mathrm{Var}(\tilde{r}_i)}_{=1}}_{\text{total risk}} = \underbrace{w_i \cdot \underbrace{\mathrm{Var}(\tilde{f}_{k(i)})}_{=1}}_{\text{systematic risk}} + \underbrace{(1-w_i) \cdot \underbrace{\mathrm{Var}(\tilde{\varepsilon}_i)}_{=1}}_{\text{unsystematic risk}} + \underbrace{2 \cdot \sqrt{w_i \cdot (1-w_i)} \cdot \underbrace{\mathrm{Cov}(\tilde{f}_{k(i)};\tilde{\varepsilon}_i)}_{=0}}_{\text{no intersection}} \quad (6)$$

The index model (5) enables a straightforward calculation of pairwise asset correlations. For two obligors i and j belonging not necessarily to different industries k(i), k(j) Eq. (3) yields:

$$\begin{aligned}\rho_{ij} &= \mathrm{Cov}\left(\tilde{r}_i;\tilde{r}_j\right) = E\left(\tilde{r}_i \cdot \tilde{r}_j\right) - E\left(\tilde{r}_i\right) \cdot E\left(\tilde{r}_j\right) \\ &= \sqrt{w_i \cdot w_j} \cdot E\left(\tilde{f}_{k(i)} \cdot \tilde{f}_{k(j)}\right) \\ &= \sqrt{w_i \cdot w_j} \cdot \rho_{k(i)k(j)} .\end{aligned} \quad (7)$$

The simplification offered by Eq. (7) is substantial: If one can obtain an (independent) estimate of the percentage portion of total asset return variance that is firm-specific for each company i = 1..n, then all $n \cdot (n - 1) / 2$ asset correlations can be calculated immediately from the $m \cdot (m - 1) / 2$ correlations between the k = 1..m industry indices. Empirical estimates of the latter $\hat{\rho}_{k(i)k(j)}$ are calculated within CreditManager™ from the last 52 historical weekly index returns[21] by means of the standard formulae[22] traditionally used in Value at Risk concepts for market risk.

Whenever all the bank's obligors in the credit portfolio under consideration are companies *listed on a stock exchange*, individual estimates of the weights w_i ($\hat{w}_i$) can be derived from the coefficient of determination R_i^2 (R-squared value) in a standard time-series regression model[23]:

$$\hat{w}_i = R_i^2 \ , \ \forall \ \ i = 1,\ldots,n . \quad (8)$$

In order to see this, recall that the coefficient of determination in a univariate, linear ordinary-least-square (OLS) regression model of the form[24]

$$(\hat{r}_{i,t} - r_{i,t})^2 = (a_i + b_i \cdot f_{k(i),t} - r_{i,t})^2 \rightarrow \min_{a,b} \quad (9)$$

is defined as:

$$R_i^2 = \frac{\sum_{t=1}^{T} (\hat{r}_{i,t} - \overline{r}_i)^2}{\sum_{t=1}^{T} (r_{i,t} - \overline{r}_i)^2} \ , \ \text{with} \ \ \overline{r}_i = \frac{1}{T}\sum_{t=1}^{T} r_{i,t} \ . \quad (10)$$

We denote the observed discrete return on stock i during the period [t – 1; t] as $r_{i,t}$ and its estimate as given by the fitted coefficients a, b in the regression equation $\hat{r}_{i,t} = a_i + b_i \cdot f_{k(i),t}$ as $\hat{r}_{i,t}$. Thus, Eq. (10) states that the coefficient of determination R_i^2 is the quotient of that part of variation of the dependent variable which is explained by the regression equation (numerator) and the total variation around its mean (denominator).

In order to clarify the important relationship between the estimated slope term in the regression equation b_i, which is a sensitivity measure resembling the beta coefficients familiar from CAPM and APT, the coefficient of determination R_i^2 and the coefficient of correlation between the stock return and its industry index $\rho_{i,k(i)}$, we transform Eq. (10) as follows[25]:

$$R_i^2 = \frac{\frac{1}{T-1} \cdot \sum_{t=1}^{T} (a_i + b_i \cdot f_{k(i),t} - \overline{r}_i)^2}{\frac{1}{T-1} \cdot \sum_{t=1}^{T} (r_{i,t} - \overline{r}_i)^2} = \frac{\hat{\mathrm{Var}}\left(a_i + b_i \cdot \tilde{f}_{k(i)}\right)}{\hat{\mathrm{Var}}\left(\tilde{r}_i\right)} = \frac{b_i^2 \cdot \hat{\mathrm{Var}}\left(\tilde{f}_{k(i)}\right)}{\hat{\mathrm{Var}}\left(\tilde{r}_i\right)}. \quad (11)$$

Considering that the sensitivity coefficient b_i can be written as[26]:

$$b_i = \frac{\hat{\mathrm{Cov}}\left(\tilde{r}_i ; \tilde{f}_{k(i)}\right)}{\hat{\mathrm{Var}}\left(\tilde{f}_{k(i)}\right)} = \frac{\hat{\rho}_{i,k(i)} \cdot \sqrt{\hat{\mathrm{Var}}\left(\tilde{r}_i\right)}}{\sqrt{\hat{\mathrm{Var}}\left(\tilde{f}_{k(i)}\right)}}, \quad (12)$$

and combining Eqs. (11) and (12), it becomes apparent that in our index model, the coefficient of determination (R-squared value) of a particular stock is simply the square of the *estimated* coefficient of correlation between the stock return and its corresponding index return:

$$R_i^2 = \hat{\rho}_{i,k(i)}^2 . \quad (13)$$

Using Eqs. (7), (8) and (13), our estimator of the asset (= equity) correlation between two companies can be expressed as:

$$\begin{aligned} \hat{\rho}_{i,j} &= \sqrt{\hat{w}_i \cdot \hat{w}_j} \cdot \hat{\rho}_{k(i),k(j)} \\ &= \hat{\rho}_{i,k(i)} \cdot \hat{\rho}_{j,k(j)} \cdot \hat{\rho}_{k(i),k(j)} . \end{aligned} \quad (14)$$

Thus, the pairwise asset correlation between two companies is decomposed into the product of the equity correlations between the two firms and their respective industry index and the correlation between these two indices.

In the case of *non-publicly traded obligor firms*, typical for many medium-sized enterprises in Germany (the so-called "Mittelstand"), the regression model described above cannot be fitted, because of a lack of stock price data. In the next section, we analyze if and how the approach currently implemented in CreditManager™ can offer a reasonable solution to this case.

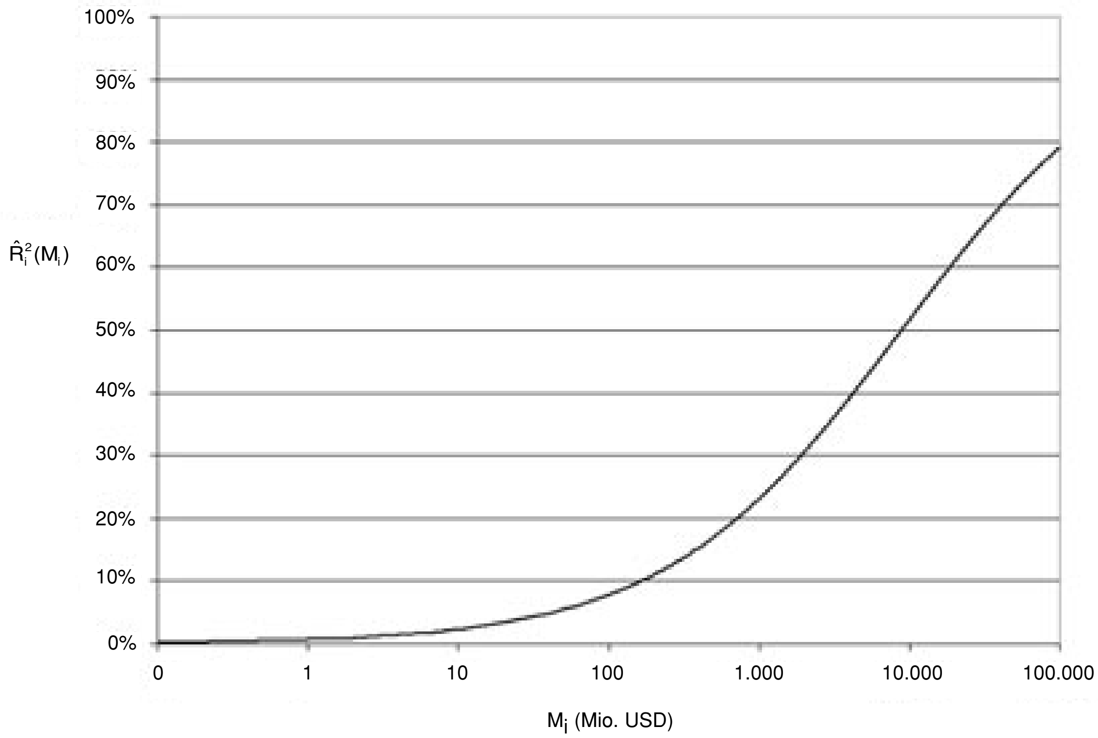
2.3 Using Company Size as a Proxy for Systematic Risk

Although not explicitly formulated in the CreditMetrics™ Technical Document, the notion that company size is an important driver of systematic risk is already anchored there: "Generally, prices for companies with large market capitalization will track the indices closely, and the idiosyncratic portion of the risk to these companies is small; on the other hand, prices for companies with less market capitalization will move more independently of the indices, and the idiosyncratic risk will be greater."[27] The Interface File Specification to CreditManager™ version 2.5 is even more precise: "In general, obligor-specific risk can be considered to be a function of company size. Larger companies have relatively small firm-specific risk because their behavior tends to be like that of the overall market (often they are components of market benchmarks). Smaller companies can have larger firm-specific risk, since they are more likely to behave independently of broad market trends and are less likely to be index components."[28]

Although the *theoretical* arguments behind the ideas expressed in the above citations are still under active discussion[29], a valid *empirical* relationship between R-squared value and company size could nonetheless provide a sound basis for calibration.[30] The current proposal offered in CreditManager™ as a potential solution to the estimation problem (8) is as follows[31]:

$$\begin{aligned} \hat{w}_i^{CM} &= \hat{R}_i^2\left(M_i\right) = \\ &= 1 - \frac{1}{1 + M_i^{\gamma} \cdot \exp(\lambda)}, \quad \forall \ i = 1, \ldots, n. \end{aligned} \quad (15)$$

Figure 2: Suggested Relationship Between the Market Value of the Firm's Total Assets (Mio. USD) and the Portion of its Systematic Risk (x-axis in logarithmic scale)



As a kind of benchmark or standard solution, Eq. (15) provides a "general rule" that gives an estimate of the overall weight of the systematic component in asset returns for each obligor depending on its size. The only input data required to calculate these weights are the market values of the firms' total assets (in USD), which we denote by M_i. The two parameters in the logistic function (15), which produces R-squared estimates between zero and one, have actually been set to $\gamma = 0{,}550$ and $\lambda = -12{,}600$ in the CreditManager™ version 3.1. A plot of the function for these parameter values is given in Figure 2.

In order to calibrate the logistic function (15), the RiskMetrics™ Group has developed its own econometric methodology, which is basically a refined quadratic programming approach.[32] The data used in their latest study dealing with this subject consists of 200 stocks from 14 nations.

3. Empirical Estimation of the Relationship Between Company Size and Systematic Risk with a Random Sample of German Stocks

3.1 Motivation and Statistical Design

The RiskMetrics™ Group (2000) benchmark study has a very broad scope, with its focus on a globally diversified credit portfolio. The sample contains only 10 German stocks (BASF, BAYER, COMMERZBANK, DAIMLER-BENZ, DEUTSCHE BANK, DRESDNER BANK, DSL HOLDING, LUFTHANSA, SIEMENS and VOLKSWAGEN) clustered in just 5 industries. Virtually all of the stocks chosen by RiskMetrics™ Group for the German sub-sample are big "*global players*", with correspondingly high market capitalizations. From our perspective, this could only lead coincidentally to a "good fit" of the parameters when applied to a portfolio of loans made to medium-sized German businesses, the typical "Mittelstand".

Because the parameter values of γ and λ were fitted in the RiskMetrics™ Group study with respect to market capitalizations denominated in USD, fluctuating *USD/EUR exchange rates* would influence company sizes measured in USD and thereby exert an unjustified and quite arbitrary influence on the obligors' shares of idiosyncratic risk.

Because there are no market capitalizations available for the large group of non-listed mid-cap firms in Germany, we decided to use the *book value of total assets* as a second, alternative measure of company size. These book values should be available in banks' internal databases and, because the regression parameters are calibrated with respect to this variable, there is no need for further adjustments using for example EBITDA-multiples in order to generate artificial "market values".

We favoured the simple, but robust econometric approach of a two-step (OLS-) regression model, which was introduced into the field of capital market research by FAMA/MACBETH (1973) and which was, starting with CHEN/ROLL/ROSS (1986), used in a number of empirical studies dealing with the derivation of risk premia from beta coefficients in the context of the APT.[33]

We wanted our calibration of the weights for the systematic components to fit in with our simplified factor model (5). Therefore, since we could not map an obligor to more than one industry, we needed an estimate of the extent to which each obligor's return was driven by that particular industry. Consequently, we had to restrict ourselves to a set of *univariate* linear time-series regressions in the first step (to calculate estimates of the coefficients of determination $\hat{R}_i^2$), which were followed in the second step by a cross-sectional regression. Moreover, because the industry correlations were estimated in CreditManager™ version 2.5 from the last 52 weekly observations, we also decided to use roughly *one year of weekly returns* for the estimation of the R-squared values.

Finally, to arrive at a reasonable "rule" relating R-squared value to company size, which can also be applied to non-publicly-traded companies, the influence of the *index weights* must be considered very carefully. In fact, the share of variation explained in the time-series regression of a stock on an index will be greater, all else equal, if the stock has a greater weighting in the index. This effect is *tautological*, insofar as the stock returns are to a greater extent self-explanatory. The problem is that this relationship is only an artificial reflex of the index composition and does not contain any usable economic meaning for our application.

3.2 Data and Results

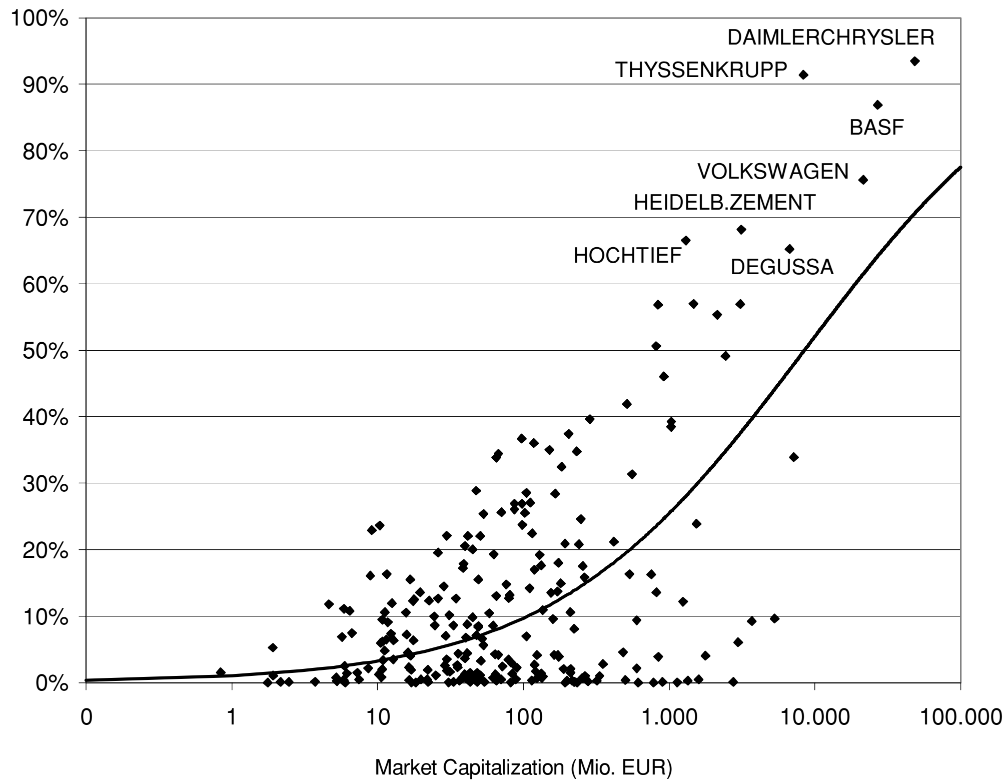
Our original sample consists of the 790 shares that were listed on the Frankfurt stock exchange on the 30.01.2002. This data was supplied by the DEUTSCHE BÖRSE AG[34] and includes the stocks' ISIN codes, company names, market capitalizations, industry affiliations and weights in the respective CDAX® industry indices. For these 790 ISIN codes, we received from BLOOMBERG weekly Friday fixing prices for the year 2001 (05.01.2001 to 25.01.2002) and the book value of total assets per 31.12.2001.

In the first step of our data selection, we removed all stocks belonging to the three industry groups of banking, insurance and financial services from the sample, because our aim was to obtain a calibration for a portfolio of corporates, not for financial intermediaries with rather atypical balance sheet characteristics. Secondly, we eliminated all stocks with missing price entries (for example because of a delisting), but we decided to retain those stocks that had a substantial portion of zero weekly returns. Zero returns are the result of illiquidity in the security, but with respect to correlation, a market which is informationally efficient would have produced joint return variations whenever they are justified. Therefore, an illiquid stock displays a constant price, because there is no trading, but the reason for a lack of trades is simply that there was no relevant new information available in the market that induced correlation.

From the remaining ISIN codes that also had a valid BLOOMBERG entry for their book value of total assets per 31.12.2001, we drew a random sample of 250 stocks. We observed that in this random sample, there were 9 companies which had issued not only common, but also preferred stock and personally registered shares. Accordingly, our final sample consists of 250 securities, but only 241 companies. The market capitalizations (book values of total assets) in our sample vary between 0,8 (4,7) and 48.445,5 (207.410,0) Mio. EUR with an average of 767,7 (2.275,3) Mio. EUR per company. The index weights have a minimum of 0,001% and a maximum of 50,931%; they are 1,792% on average. Our sample covers all targeted 16 CDAX® industry indices and represents more than one quarter of the total market capitalization of all listed companies in these industries.

In order to estimate the coefficient of determination (R-squared) for each of the 241 companies, we also downloaded from BLOOMBERG, the time series of the 16 indices for the 56 Fridays selected. The R-squared values were calculated as described in section 2.2 using 55 discrete weekly returns of each stock i and its corresponding industry index k(i). Because the index values were

Figure 3: Scatter Plot of Coefficients of Determination (R-squared) Against Market Capitalizations (x-axis in logarithmic scale)



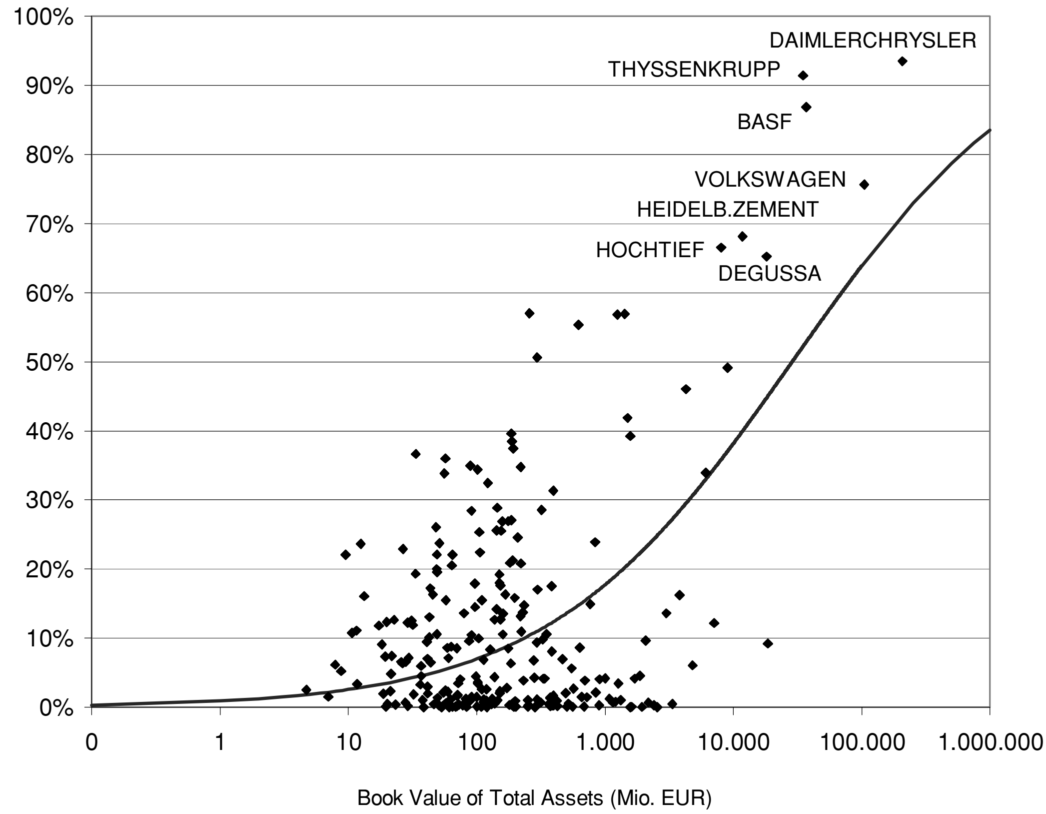
calculated by DEUTSCHE BÖRSE AG on the basis of closing prices (at about 20.15 h), there is a time lag of approximately 8 hours to the corresponding stock prices which were fixed at 12.30 h. This lag certainly led to an overall downward bias in our estimates of the coefficients of determination. However, as long as this bias can be assumed as of nearly equal size for all stocks, the error was probably cancelled out in the cross-sectional regressions of our second step.

For the nine companies with more than one class of stock, we calculated one R-squared value for each security and use the market-capitalization-weighted average for the company. For example, in the case of VOLKSWAGEN AG, we calculated an R-squared of 80,9139% for the common (ISIN code DE0007664005) and of 75,6346% for the preferred stock (ISIN code DE0007664039). With given market capitalizations of 17.684,9 Mio. EUR for the common and 3.787,5 Mio. EUR for the preferred shares, we arrive at an R-squared (with respect to the CDAX® Automobile index) of 79,9827% for the company.

In our sample, the coefficients of determination which result from the first-step-regressions, vary between 0,0001% and 93,5022% with an average of 12,4740%. The scatter plot in Figure 3 reveals the manner in which the 241 calculated R-squared values are related to the firms´ market capitalizations. From a first visual inspection, it is obvious that a logistic function of the type given in Eq. (15) could well be used to fit the data.

Figure 4 shows the relationship between the firms´ R-squared estimates and our second proxy for

Figure 4: Scatter Plot of Coefficients of Determination (R-squared) Against Book Value of Total Assets (x-axis in logarithmic scale)



company size, namely the book value of their total assets. It is evident that the relationship seems rather similar.

Regressing all the 241 R-squared values on our two measures of company size leads to the estimates of parameter values in the logistic function (15) that are displayed in Table 1.

As can be seen from Figs. 3 and 4, both size variables undoubtedly explain a significant portion of the cross-sectional variation in the "true" R-squared values. In the nonlinear cross-sectional regressions summarized in Table 1, all four coefficient signs are significant at the 5% level (at minimum). Surprisingly, the difference caused by the change of proxy (independent variable) seems rather small and, furthermore, our estimates for the parameters γ and λ are quite similar to those obtained in the above-mentioned RiskMetrics™ Group study, despite the many differences in methodology and data. On the whole, one could be tempted to say, that, based on these results, the calibration of (15) currently implemented in CreditManager™ should work rather well with German medium-sized obligors, even if EUR-book values of total assets are entered instead of USD-market capitalizations.

However, two observations should lead to a less enthusiastic interpretation of these results: Firstly, there is a relatively small number of *outliers*. The names of the most prominent outliers are revealed in Figs. 3 and 4. These companies, whose R-squared values are extremely high, also have particularly high weights in their CDAX® industry indices. Consequently, the use of a size variable as a proxy for asset correlation is somewhat questionable here, since the high R-squared values may simply be due to the above-mentioned tautology. Secondly, the use of book values instead of market capitalizations considerably lowers the percentage of variation in the dependent variable explained in the cross-sectional regression, namely from 40,714% to 29,587%. Comparing the correlations between index weights and book values (66,712%) on the one hand and between index weights and market capitalizations (75,558%) on the other, gives rise to the suspicion that the higher explanatory power of market capitalizations derives from the trivial fact that a stock's market value is merely a better proxy for its index weight than its book value.

In order to investigate further the potentially distorting role of the index weights, we split up our total sample in two sub-samples and compare the results. The first (second) sub-sample contains the 121 (120) companies with higher (lower) market capitalizations. In particular, all those companies with extremely high index weights, like THYSSENKRUPP (50,931% in Basic Resources), DAIMLERCHRYSLER (43,984% in Automobile), BASF (41,150% in Chemicals), HEIDELB.ZEMENT (34,760% in Construction) and VOLKSWAGEN (19,495% in Automobile),

Table 1: Nonlinear Cross-Sectional Regression Summary Statistics; Complete Sample (241 Firms)

Parameter	γ			λ			Percentage of variation explained in the cross-sectional regression
Independent variable	***Estimate***	95%-Lower Bound	95%-Upper Bound	***Estimate***	95%-Lower Bound	95%-Upper Bound	
Market capitalization	***0,503***	0,418	0,588	***–11,490***	–13,224	–9,756	40,714%
Book value of total assets	***0,457***	0,369	0,545	***–11,009***	–12,857	–9,162	29,587%

Table 2: Average Index Weights, Market Capitalizations, Book Values of Total Assets and Coefficients of Determination (R-squared) for the two Sub-Samples

	Index Weight	Market Capitalization (Mio. EUR)	Book Value of Total Assets (Mio. EUR)	R-squared
			Sub-Sample 1	
Average	3,446%	1.503,0	4.411,2	18,1379%
Max	50,931%	48.445,5	207.410,0	93,5022%
Min	0,069%	64,1	33,5	0,0001%
			Sub-Sample 2	
Average	0,125%	26,4	121,6	6,7630%
Max	0,664%	64,0	1.609,7	28,8702%
Min	0,001%	0,8	4,7	0,0002%

which have already been characterized as outliers before, are included in the first sub-sample, which has an average company index weight of 3,446%.

As can be seen from Table 2, the average index weight in the second sub-sample is only 0,125% with the maximum weight of a single company in its industry index being less than 1%. Therefore, we can reasonably assume that the tautological effect that leads to a higher R-squared estimate for a stock, simply because it has a higher index weight, is filtered out successfully in the second sub-sample. This accomplishment is highlighted additionally by the fact that the correlation between the variables index weight and R-squared, drops from 64,882% in the first sub-sample to –24,348% in the second. Moreover, the average book value of total assets of about 121,6 Mio. EUR for the companies in the second sub-sample (in contrast to 4.411,2 Mio. EUR in the first) indicates that this sub-sample is, by its very nature, far better suited to infering calibration results with respect to medium sized companies. The regression summary statistics for the two sub-samples are given in Tabs. 3 and 4.

While the results for the first sub-sample that contains the "large" companies presented in Table 3 are in line with our results for the complete sample (Table 1) and with those of the RiskMetrics™ Group´s benchmark study, the results for the second sub-sample are *not*. The exclusion of the firms with high index weights, particularly the above-mentioned outliers, removes most of the explanatory power from the cross-sectional regression. For the group of 120 "smaller" firms, both our proxies for company size perform very poorly. They explain only 3,608% and 3,649% respectively of the cross-sectional variation in the R-squared values. Even now, the book value

Table 3: Nonlinear Cross-Sectional Regression Summary Statistics: Sub-Sample 1 (121 Firms with Higher Market Capitalizations)

Parameter	γ			λ			Percentage of variation explained in the cross-sectional regression
Independent variable	***Estimate***	95%-Lower Bound	95%-Upper Bound	***Estimate***	95%-Lower Bound	95%-Upper Bound	
Market capitalization	***0,571***	0,423	0,719	***–12,956***	–16,043	–9,869	37,537%
Book value of total assets	***0,465***	0,328	0,602	***–11,118***	–14,053	–8,182	27,198%

Table 4: Nonlinear Cross-Sectional Regression Summary Statistics: Sub-Sample 2 (120 Firms with Lower Market Capitalizations)

Parameter	γ			λ			Percentage of variation explained in the cross-sectional regression
Independent variable	***Estimate***	95%-Lower Bound	95%-Upper Bound	***Estimate***	95%-Lower Bound	95%-Upper Bound	
Market capitalization	***0,233***	–0,021	0,487	***–6,549***	–10,884	–2,215	3,608%
Book value of total assets	***-0,184***	–0,373	0,005	***0,671***	–2,659	4,002	3,649%

variable does slightly better than market capitalization, but the estimated coefficients bears the "wrong" signs, indicating that R-squared would decrease with an increasing book value of total assets. This is probably due to the above-mentioned inverse relationship between book value of total assets and index weight, which is the "true" driver behind R-squared. The estimated parameters for market capitalization bear the "correct" sign, but the direction of influence on R-squared is not significant at the 5% level, because the sign of γ may change to negative. Hence, the alleged robustness of calibration results is thoroughly shaken and these results call into question the entire concept of transferring a valid empirical relationship between company size and the portion of idiosyncratic risk to the case of non-listed obligors.
In order to gain more confidence in these results and to further support our argument that the positive relationship between a company's size and its systematic risk is not valid beyond "large" liquidly traded stocks which directly influence the indices to a considerable extent, we have carried out another four *linear* cross-sectional OLS-regressions with the companies´ R-squared value as dependent variable. This is to test whether the sign of the beta coefficient differs significantly from zero. Our results are presented in Tabs. 5 and 6.
Using the first sub-sample, the sign of the slope b (as well as the sign of the constant a) is significantly positive, even at a confidence level of 99% for both proxies of company size. The independent variables explain 33,5% (26,9%) of the variation in systematic risk. This apparently positive relationship between size and the extent of systematic risk disappears completely again, if the second sub-sample is evaluated:

Table 5: Linear Cross-Sectional Regression Summary Statistics: Sub-Sample 1 (121 Firms with Higher Market Capitalizations)

Parameter	a (constant)			b (slope)			Percentage of variation explained in the cross-sectional regression
Independent variable	***Estimate***	t-test statistic	significance	***Estimate***	t-test statistic	significance	
Market capitalization	***0,147***	8,870	0,000	**0.00002286**	7,749	0,000	33,5%
Book value of total assets	***0,158***	9,271	0,000	***0,00000519***	6,625	0,000	26,9%

Table 6: Linear Cross-Sectional Regression Summary Statistics: Sub-Sample 2 (120 Firms with Lower Market Capitalizations)

Parameter	a (constant)			b (slope)			Percentage of variation explained in the cross-sectional regression
Independent variable	***Estimate***	t-test statistic	significance	***Estimate***	t-test statistic	significance	
Market capitalization	***0,051***	4,484	0,000	***0,001***	1,764	0,080	2,6%
Book value of total assets	***0,075***	10,435	0,000	***–0,00005981***	–2,007	0,047	3,3%

The percentage of variation in the 120 time-series-regression R-squared values, which is explained in the cross-sectional regressions, is only 2,6% (3,3%). The estimated slope term for market capitalization bears the “correct” sign, but does not differ from zero significantly (at the 5% level). The estimated slope term for book value of total assets is significantly different from zero (confidence level 4,7%), but again bears the “wrong” sign.

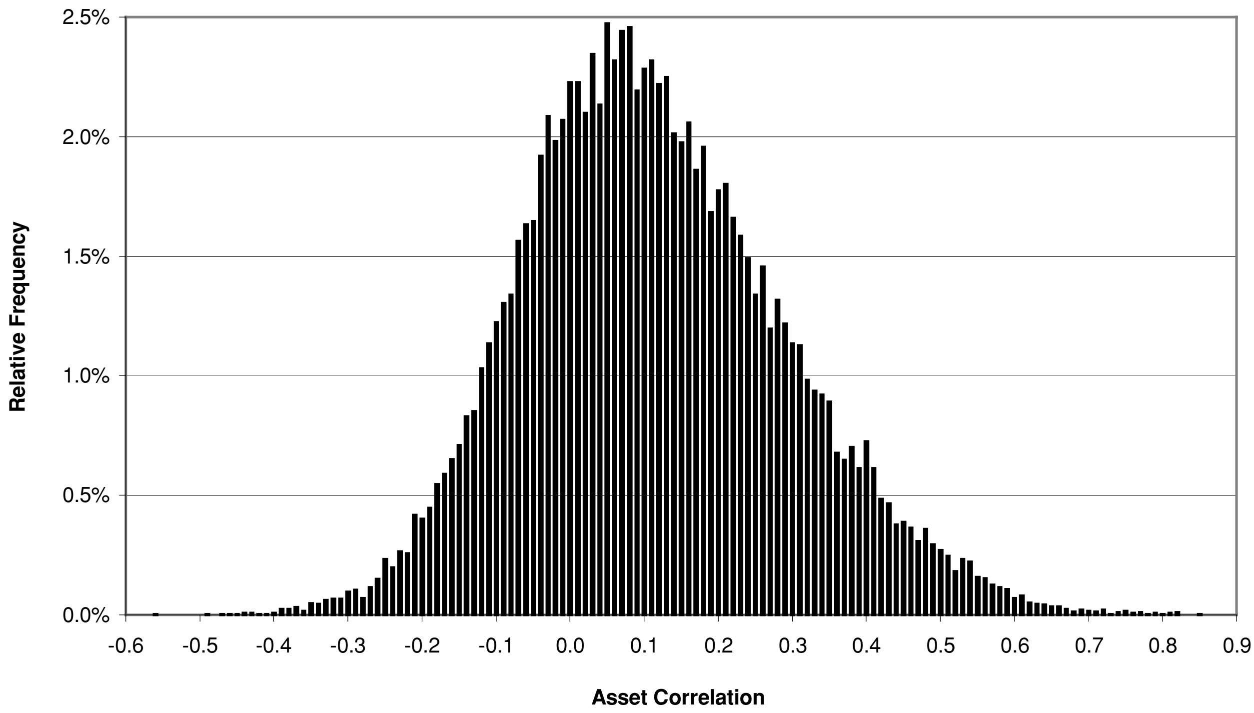
4. Comparison of Four Alternative Estimation Models for Pairwise Asset Correlations

In order to investigate further under which circumstances company size serves as a good proxy for asset (equity return) correlation, we use the results presented in section 3 and compare the in-sample, cross-sectional explanatory power of four different estimation models (M1, M2a, M2b and M3) for both the complete sample and the two sub-samples. The question we wish to answer here is: does the inclusion of one of our two size proxies improve the predictive power of the index model and if so, under what conditions?

Firstly, we calculated the 28.920 pairwise equity return correlations for all 241 companies. In the special cases where there was more than one type of stock issued, we constructed a new equity return time series for each company: the market-capitalization-weighted average of the return time series for the different types of stock. Figure 5 shows the histogram of asset correlations across the firms in our sample. The distribution is not perfectly symmetrical, but is characterized by a slightly “fatter” upper tail. Moreover, 28,496% of the calculated correlations are negative. The distribution has a mean (maximum, minimum) of 10,743% (84,688%, –56,767%). Concerning the two sub-samples, the average asset correlation among the “larger” (“smaller”) firms is 13,420% (9,335%). Notably, the overall average asset correlation of about 10% which we inferred from 55 weekly observations of equity returns of 241 German companies during 2001, the year including September 11th, is *much lower* than the range of 20% to 35% cited in the CreditMetrics^{TM} Technical Document.[35] It is also much lower than the 20% reported as the average of current industry practice by KOYLUOGLU/WILSON/YAGUE (2003).[36] However, our average asset correlation is not far from the lower bound of 12% that is assumed in the New Basel Accord.[37]

Secondly, we estimated four different versions of the CreditMetrics^{TM} index model in order to evaluate their explanatory power as to the cross-sectional variation in the given asset correlations (in-sample-test). These four models all rely on the same estimated CDAX® index correlations, which were calculated from their 56 weekly Friday clos-

Figure 5: Histogram of 28.920 Pairwise Asset Correlations



ing prices from 05.01.2001 to 25.01.2002 and which are given in Table 7. Since all these index correlations are positive (minimum 15,746%, maximum 81,654%, average 57,848%), it is clear that none of the four models is able to explain the considerable share of negative asset correlations in the sample.

The first model (M1), which we use to empirically estimate pair wise asset correlations, is that given in Eq. (14). This calibration, which can only be implemented if the time series of stock prices are available that would also allow a direct estimation of pair wise asset correlation (which might be computationally too burdensome for larger portfolios) makes use of the "true" empirical R-squared values for each company. The model is therefore not applicable to non-listed firms, but it provides an indication as to the maximum explanatory power that could be achieved if all the data needed to estimate asset correlations by means of the CreditMetrics™ index approach was available.

The model M2a is based on the approach originally proposed by the RiskMetrics™ Group (see Eq. (15) again) and relates systematic risk to company size in terms of market capitalization, whereas model M2b uses the firms´ book values of total assets as a proxy for company size. In particular, model M2b could easily be implemented for a portfolio of medium-sized, non-listed obligor firms, as the information needed (each obligor's book value of total assets and its industry affiliation) should be readily available in banks´ internal databases. Further, both models M2a and M2b reveal significantly reduced data requirements in

Table 7: Estimated CDAX® Industry Index Return Correlations, Calculated from 56 Weekly Friday Closing Prices (05.01.2001 to 25.01.2002)

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	Automobile	1,000	0,793	0,730	0,573	0,762	0,474	0,702	0,621	0,752	0,690	0,647	0,444	0,635	0,760	0,532	0,382	0,422	0,655	0,793
2	Banks	0,793	1,000	0,742	0,552	0,759	0,612	0,703	0,768	0,719	0,788	0,724	0,572	0,757	0,797	0,593	0,424	0,632	0,805	0,716
3	Chemicals	0,730	0,742	1,000	0,456	0,746	0,504	0,602	0,661	0,743	0,773	0,720	0,522	0,696	0,731	0,464	0,347	0,589	0,638	0,727
4	Media	0,573	0,552	0,456	1,000	0,518	*0,317*	0,724	0,345	0,541	0,560	0,571	*0,330*	0,432	0,515	0,604	0,436	*0,222*	0,608	0,478
5	Basic Resources	0,762	0,759	0,746	0,518	1,000	0,486	0,695	0,578	0,800	0,772	0,691	0,597	0,622	0,768	0,561	0,342	0,457	0,641	0,794
6	Food&Beverages	0,474	0,612	0,504	*0,317*	0,486	1,000	0,512	0,515	0,491	0,623	0,713	0,469	0,495	0,517	0,548	*0,171*	0,480	0,652	0,541
7	Technology	0,702	0,703	0,602	0,724	0,695	0,512	1,000	0,483	0,637	0,588	0,612	0,405	0,465	0,641	0,691	0,559	*0,292*	0,702	0,639
8	Insurance	0,621	0,768	0,661	0,345	0,578	0,515	0,483	1,000	0,691	0,589	0,642	0,578	0,743	0,643	0,427	*0,284*	0,501	0,657	0,608
9	Transp. & Logistics	0,752	0,719	0,743	0,541	0,800	0,491	0,637	0,691	1,000	0,696	0,613	0,475	0,541	0,659	0,556	0,338	0,486	0,715	0,755
10	Machinery	0,690	0,788	0,773	0,560	0,772	0,623	0,588	0,589	0,696	1,000	0,817	0,652	0,671	0,816	0,555	*0,161*	0,648	0,683	0,750
11	Industrial	0,647	0,724	0,720	0,571	0,691	0,713	0,612	0,642	0,613	0,817	1,000	0,667	0,712	0,708	0,531	*0,196*	0,507	0,629	0,697
12	Construction	0,444	0,572	0,522	*0,330*	0,597	0,469	0,405	0,578	0,475	0,652	0,667	1,000	0,515	0,610	0,368	*0,157*	0,437	0,426	0,623
13	Pharma & Health	0,635	0,757	0,696	0,432	0,622	0,495	0,465	0,743	0,541	0,671	0,712	0,515	1,000	0,720	0,487	*0,285*	0,589	0,614	0,575
14	Retail	0,760	0,797	0,731	0,515	0,768	0,517	0,641	0,643	0,659	0,816	0,708	0,610	0,720	1,000	0,593	*0,262*	0,607	0,660	0,742
15	Software	0,532	0,593	0,464	0,604	0,561	0,548	0,691	0,427	0,556	0,555	0,531	0,368	0,487	0,593	1,000	0,451	0,371	0,689	0,556
16	Telecommunications	0,382	0,424	0,347	0,436	0,342	*0,171*	0,559	*0,284*	0,338	*0,161*	*0,196*	*0,157*	*0,285*	*0,262*	0,451	1,000	*0,197*	0,436	*0,225*
17	Utilities	0,422	0,632	0,589	*0,222*	0,457	0,480	*0,292*	0,501	0,486	0,648	0,507	0,437	0,589	0,607	0,371	*0,197*	1,000	0,593	0,503
18	Financial services	0,655	0,805	0,638	0,608	0,641	0,652	0,702	0,657	0,715	0,683	0,629	0,426	0,614	0,660	0,689	0,436	0,593	1,000	0,643
19	Consumer-cyclical	0,793	0,716	0,727	0,478	0,794	0,541	0,639	0,608	0,755	0,750	0,697	0,623	0,575	0,742	0,556	*0,225*	0,503	0,643	1,000

Table 8: Methodology and Parameter Settings of the Four Estimation Models

Model	Differences in R-squared Estimation			
	Size Proxy	*Complete Sample*	*Sub-Sample 1*	*Sub-Sample 2*
M1	No	Individual Time-Series Regression per Company		
M2	Yes	Individual Estimation per Company according to Eq. (15) with		
M2a	Market Capitalization	γ = 0,503, λ = –11,490	γ = 0,571, λ = –12,956	γ = 0,233, λ = –6,549
M2b	Book Value of Total Assets	γ = 0,457, λ = –11,009	γ = 0,465, λ = –11,118	γ = –0,184, λ = 0,671
M3	No	Average R-squared per Sample		
		12,4740%	18,1379%	6,7630%

comparison to model M1, because no time series data for single share prices are involved.

Model M3 represents the simplest approach: the model does not contain a proxy for size, but uses the same average R-squared estimate of the sample across all obligors. This approach assumes that systematic risk is constant within a homogeneous group of borrowers, for example across the "smaller" companies in sub-sample 2. It seems significant that with model M3, the choice of the average R-squared is highly relevant for Credit Value at Risk calculations, but that its absolute value does not play a role in assessing the model's explanatory power with respect to asset correlations. The only economic content in model M3 that is used to explain the cross-sectional variation in asset correlations lies in the CDAX® industry index correlations and in the assignment of company pairs to them. In this sense, model M3 is a kind of minimum-requirement-version of the CreditMetrics™ index approach.

A synopsis of the differences in the estimation of systematic risk between the four model specifications is given in Table 8.

Our comparison of the estimation performance of the four models (in-sample-test) is based on regressing the predicted 28.920 (7.260; 7.140) pair wise asset correlations of the complete sample (sub-sample 1, sub-sample 2) on their observed "true" values. A summary of the results of the 12 linear cross-sectional OLS-regressions, which comprises the percentage of cross-sectional variation in asset correlation explained by the model, the overall F-test statistic and the corresponding empirical level of significance, is presented in Table 9.

As can be concluded from the overall F-test statistics, all four models are significant at the 1% level. Model M1 has the best estimation performance, which is not surprising as it makes use of the

Table 9: Linear Cross-Sectional Regression Results for the Four Estimation Models

Model	In-Sample Test of Estimation Performance		
	Complete Sample	*Sub-Sample 1*	*Sub-Sample 2*
M1	31,7%[a]; 13.451,057***[b]	43,9%; 5.689,879***	29,0%; 2.911,046***
M2a	4,8%; 1.456,731***	6,6%; 516,621***	9,4%; 737,989***
M2b	1,3%; 371,545***	2,1%; 154,324***	12,3%; 1001,332***
M3	5,7%; 1.755,550***	4,0%; 300,667***	10,0%; 794,914***
Number of Observations	28.920	7.260	7.140

a) Percentage of cross-sectional variation in asset correlation explained by the model

b) F-test statistic with empirical significance (*** represents 1% level) "true" R-squared values. For sub-sample 1 with the "larger" stocks, model M1 can explain as much as 43,9% of variation in the dependent variable. For sub-sample 2 containing the "smaller" stocks, 29,0% of cross-sectional variation can still be attributed to the industry factors and individual R-squared values. The plain index model M3, which does not make use of information beyond the correlations between the CDAX® industry indices, performs better for the "smaller" (10,0% of cross-sectional variation are explained) than for the "bigger" (just 4,0% of cross-sectional variation are explained) companies. Interestingly, the use of market capitalization as a size proxy in model M2a does *not* improve the predictive power of the CreditMetrics™ index approach for the complete sample (4,8% in comparison to 5,7%) nor for sub-sample 2 (9,4% versus 10,0%), but *only* for the sub-sample containing the "bigger" stocks (6,6% in comparison to 4,0%). Hence, there seems to be an influence of company size in terms of market capitalization on asset correlation that holds beyond the tautological impact via index weights, but that is restricted to "larger" stocks. Model M2b, which uses the book value of total assets as proxy variable, yields the worst performance of the four specifications, explaining only 1,3% (2,1%) of the variation in asset correlations in the complete sample (sub-sample 1). Only for sub-sample 2 is the predictive power much higher at 12,3%, but, as is already known from section 3, the regression coefficient bears the "wrong" sign, indicating that both asset correlations and R-squared values would decrease here with an increase in total assets (see the grey shaded areas in Tabs. 7 and 8).

5. Conclusions and Outlook

This paper presents the first empirical evidence on the problem of calibrating the CreditMetrics™ asset correlation concept for a German mid-cap loan portfolio. Our findings on the relationship between a firm's size and its systematic risk, defined as the percentage movement of a stock explained by its corresponding CDAX® industry index (R-squared), are as follows. With respect to the complete sample of 241 companies, our parameter estimates are quite similar to those obtained in the RiskMetrics™ Group (2000) benchmark study, despite the many differences in methodology and data. However, this alleged robustness of calibration results is very much called into question, if we split up our sample in two sub-samples according to the companies´ market capitalizations. While the apparently positive relationship between obligor size and the extent of its systematic risk remains quite stable in the first sub-sample which contains all the "heavy weight" firms that constitute material portions of its CDAX® industry indices, it disappears *completely* in the second sub-sample. Furthermore, the use of the variable book value of total assets as an alternative proxy for company size that is readily available in banks also for non-listed obligor firms, is generally inferior to market capitalization. Both these findings suggest that a positive relationship between a stock's R-squared and the firm's size is induced solely by its index weight and is thus tautological. In order to investigate whether there is a significant influence of company size on asset correlation in our data that is not induced by the firms´ index weights, we compared four different versions of the CreditMetrics™ index model in terms of their in-sample explanatory power to predict pairwise asset correlations: While the proxy variable book value of total assets does not seem to contain any useful, unambiguous information about the extent of existing correlations, the inclusion of the variable market capitalization does improve the predictive power of the index model slightly, but only with respect to "bigger" companies. Therefore, according to our empirical results, model M3 with an average R-squared of about 7%, corresponding to an average asset correlation of about 9%, seems to be a reasonable starting point for Credit Value at Risk calculations with a portfolio of German medium-sized corporate obligors in CreditMetrics™. Although this recommendation leads to an average asset correlation that is less than half of the lower bound of the range originally suggested in the CreditMetrics™ Technical Document, it is still much higher than the estimates recently derived by HAMERLE/LIEBIG/RÖSCH (2003) and DÜLLMANN/SCHEULE (2003) for the Basel II single-factor model on the basis of historical default rate data. This discrepancy highlights the fact that a well-established empirically valid link between equity (return) correlations and actual default correlations is still missing. In order to close the parameterization gap between MERTON-style credit portfolio models and the approaches based on historical default time series, more academic research along the lines of HAMERLE/RÖSCH (2004) is needed. Only further empirical results can finally justify the use of stock market data to calibrate internal credit risk models and thereby reconcile the apparent differences between the competing modelling frameworks.

FOOTNOTES

[1] Cf. SCHÖNBUCHER (2000), p. 4.

[2] See the Technical Documents by GUPTON/FINGER/BHATIA (1997) and CREDIT SUISSE FIRST BOSTON (1997).

[3] See the overview in BASEL COMMITTEE ON BANKING SUPERVISION (1999) and in BLUHM/OVERBECK/WAGNER (2003). Cf. also SAUNDERS/ALLEN (2002).

[4] See KEALHOFER/BOHN (2001).

[5] See BASEL COMMITTEE ON BANKING SUPERVISION (1999), p. 32.

[6] A conceptual comparison of the above-mentioned credit portfolio models and their application to German middle market loan portfolios is given by KERN/RUDOLPH (2001).

[7] See RiskMetrics™ Group (2000).

[8] Note that the model used by DIETSCH/PETEY (2002), p. 307 f. for SME loan portfolios can be regarded as a special case of our suggestion.

[9] See CAREY/HRYCAY (2001) for the parameterization of credit risk models with rating data.

[10] Cf. GUPTON/FINGER/BHATIA (1997), p. 87 f., footnotes 3 and 4. See also KEALHOFER/BOHN (2001), p. 2 ff., who summarize the application of option pricing theory to default risk.

[11] See e. g. CROUHY/GALAI/MARK (2000), p. 76 f.

[12] Cf. EMBRECHTS/MCNEIL/STRAUMANN (1999) and FREY/MCNEIL/NYFELER (2001). See also HAHNENSTEIN/RÖDER (2003), who discuss some limitations of the concept of correlation with respect to corporate hedging.

[13] Cf. GUPTON/FINGER/BHATIA (1997), p. 89, Eq. (8.5). See also OVERBECK/STAHL (2003) for the relationship between asset and default correlations. Empirical evidence of default correlation is provided by DE SERVIGNY/RENAULT (2003) for the US and by HAMERLE/RÖSCH (2003) and RÖSCH (2003) for Germany.

[14] See KEALHOFER/BOHN (2001), p. 11 f. See also LI (2000), p. 49 f., particularly Eqs. (10) and (11). BLUHM/OVERBECK/WAGNER (2003), p. 103 ff., provide an introduction to the use of copulae in credit risk measurement.

[15] See BLUHM/OVERBECK/WAGNER (2003), especially p. 71, Eqs. (2.34) and (2.35).

[16] Cf. GUPTON/FINGER/BHATIA (1997), p. 93. The problem of using *equity* correlations as a proxy for *asset* correlations, which has already been recognized as a potential drawback by the model's inventors themselves and which has recently been attacked on theoretical and empirical grounds by ZENG/ZHANG (2002) of KMV, does *not* form the focus of our paper. See also SCHÖNBUCHER (2000) for an introduction to factor models in credit risk measurement.

[17] Cf. GUPTON/FINGER/BHATIA (1997), p. 98.

[18] Cf. GUPTON/FINGER/BHATIA (1997), p. 94 – 96 and additionally Appendix I, p. 166. The 10 CDAX® indices they provide have been amended in the meantime to the 19 CDAX® sub-indices calculated by the DEUTSCHE BÖRSE AG (2001). Cf. DEUTSCHE BÖRSE AG (2001), p. 4.

[19] Note that this definition of systematic and unsystematic return components does *not* necessarily imply that the systematic part is the one that cannot be deleted through appropriate diversification. Therefore, our definition differs from that used in traditional portfolio theory and in the CAPM. Moreover, the definition used here is neither identical to that of the APT according to ROSS (1976, 1977) which defines systematic risk in terms of *unexpected* changes of some factors and does not identify these factors explicitly as industry indices. Finally, the APT risk factors are assumed to be (almost) uncorrelated which is obviously not true for the equity indices used in CreditMetrics™, where their differing correlations represent a part of the crucial and most valuable input data. PESARAN/SCHUERMANN/TREUTLER/WEINER (2003) propose a credit portfolio model that is related explicitly to the APT framework.

[20] Because we are solely interested in estimating the pairwise asset return correlations, we can assume standardized company (asset) returns as well as standardized (equity) index returns and idiosyncratic returns according to Eq. (5) without loss of generality.

[21] Note that this sample size differs significantly from the 190 observations mentioned in the Technical Document, p. 97. Furthermore, in CreditManager™ version 2.5, the index values were transformed into discrete and not log returns. With the introduction of CreditManager™ version 3.1, the estimation of asset correlations can no longer be duplicated in detail by users. The CDAX® index set has been replaced by one from MSCI and the index time series actually used do not lay open, because of missing redistribution rights.

[22] Cf. GUPTON/FINGER/BHATIA (1997), p. 97, Eqs. [8.7] to [8.9]. These formulae provide unbiased estimators for variances and covariances, if the time series observations are interpreted as independent drawings from the unchanged "true" distributions ("i. i. d. property").

[23] Note the criticism from KEALHOFER/BOHN (2001), p. 12.

[24] See e. g. COPELAND/WESTON (1992), Appendix C, p. 877–893, for a brief overview of regression analysis.

[25] SEE BLUHM/OVERBECK/WAGNER (2003), p. 45, Eq. (1.19) for a similar result in the context of the KMV model.

[26] Cf. COPELAND/WESTON (1992), p. 879, Eq. (C.3) and p. 881, footnote 5.

[27] GUPTON/FINGER/BHATIA (1997), p. 98.

[28] RiskMetrics™ Group (1999), p. 54.

[29] See DÜLLMANN/SCHEULE (2003), p. 4 f. including further references.

[30] Recently, a growing number of empirical studies has dealt with this relationship with respect to the New Basel Accord. Cf. DIETSCH/PETEY (2004), LOPEZ (2002) and DÜLLMANN/SCHEULE (2003). Note that these results are restricted to the Basel one-factor model and are not applicable to the more general CreditMetrics™ multi-factor framework. For the KMV model see also the calibration proposal by PITTS (2004), especially p. 77.

[31] Cf. RiskMetrics™ Group (2000), p. 20, especially the rearrangement of Eq. (C.3). See also again RiskMetrics™ Group (1999), p. 54 and RiskMetrics™ Group (2002), p. 3.

[32] See RiskMetrics™ Group (2000) for further details.

[33] Cf. HAHNENSTEIN/LOCKERT (2001), p. 603 ff.

[34] Available from ip.hotline@deutsche-boerse.com.

[35] Cf. GUPTON/FINGER/BHATIA (1997), p. 93, especially footnote 8.

[36] KOYLUOGLU/WILSON/YAGUE (2003), p. 13: "In practice, direct asset correlation estimates are usually in the range of 10% to 30%, averaging about 20%, and vary by rating and size."

[37] See p. 50 in BASEL COMMITTEE ON BANKING SUPERVISION (2003).

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